

Title: Trigonometric Solutions to a Dead Reckoning Air Navigation Problem Using Vector Analysis and Advanced Organizers

Brief Overview:

We will complete a dead reckoning navigation problem following certain steps. Dead reckoning means flying a compass heading from a check point for a predetermined period of time. The basic steps of dead reckoning are:

- Draw a straight line between the check points selected.
- Find the true course: Measure the angle between true north and the course line drawn.
- Convert the true course to a compass heading that we can fly using cockpit instruments, e.g. the magnetic compass. We do this by correcting for wind drift, magnetic variation (the difference between true north and magnetic north), and compass deviation.
- Determine the speed we will make good over the ground and the time the flight will take.

Links to NCTM Standards:

- **Mathematics as Problem Solving**

This learning unit presents a basic problem in air navigation and outlines the mathematics needed to solve it.

- **Mathematics as Communication**

At the conclusion of instruction students will describe what they have learned in a presentation to the staff and student body.

- **Mathematics as Reasoning**

Reasoning skills, in particular geometric reasoning skills are required during the computations to ensure that intermediate answers are reasonable and correct.

- **Mathematical Connections**

The material taught in this unit is immediately connected with both geometry and vector analysis. The computation of wind corrections in particular is a direct application of analytic geometry to aviation.

- **Computation and Estimation**

This learning unit requires and exercises computational skills in computing the various values required for the flight plan.

- **Geometry**

Most of the computations required are done as geometric computations, either computing angles (courses) or distances. Geometric manipulations are central to this learning unit.

- **Measurement**

Measurement skills are exercised both in recovering distance measurements from the chart and in using the manual flight computer.

- **Geometry from an Algebraic Perspective**

The computation of wind corrections is an exercise in analytic geometry, in particular vector computations.

Grade/Level:

This learning unit is appropriate for grades 9 through 12.

Duration/Length:

This learning unit is designed to be presented in a period of approximately three hours.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Working with angles
- Vector addition

Objectives:

Students will:

- plan a flight using dead reckoning methods and a manual flight computer.
- explain the meanings of wind corrections, magnetic variation, and magnetic deviation.

Materials/Resources/Printed Materials:

- Air Navigation Chart - Charts are available free of charge from the State Aviation Administration.
- Protractor or PN-1 plotter.
- CSG flight navigation calculator - A simple calculating tool, similar to a circular slide rule.²⁴⁶ Inexpensive cardboard models can be purchased and student copies can be made by reproducing the calculator parts. (Note: If calculators are not available, the same computations can be done with paper, protractor, and ruler.)
- Flight Log - Available at very low cost from pilot supply stores or from the fixed base operator at any local airport.

Development/Procedures:

- 1) The instructor will describe the navigation planning process and work an example before the class.
- 2) Students will work an example and present their answers to the class.
- 3) A solution will be selected that will be flown in a simulator during a later learning unit.

Performance Assessment:

Performance will be assessed informally during the period when students are working their examples. The instructors will examine the students work and will correct errors as they occur and are found. This learning unit is not currently constructed to provide formal evaluation of student performance.

Extension/Follow Up:

This learning unit can be followed by a unit in which a flight is planned using these procedures and then is flown in a simulator.

Authors:

Bill Solms
University of Maryland Eastern Shore

Phil Wiest
St Vincent Pallotti HS
Prince George's County, MD

JEPPESSEN SANDERSON

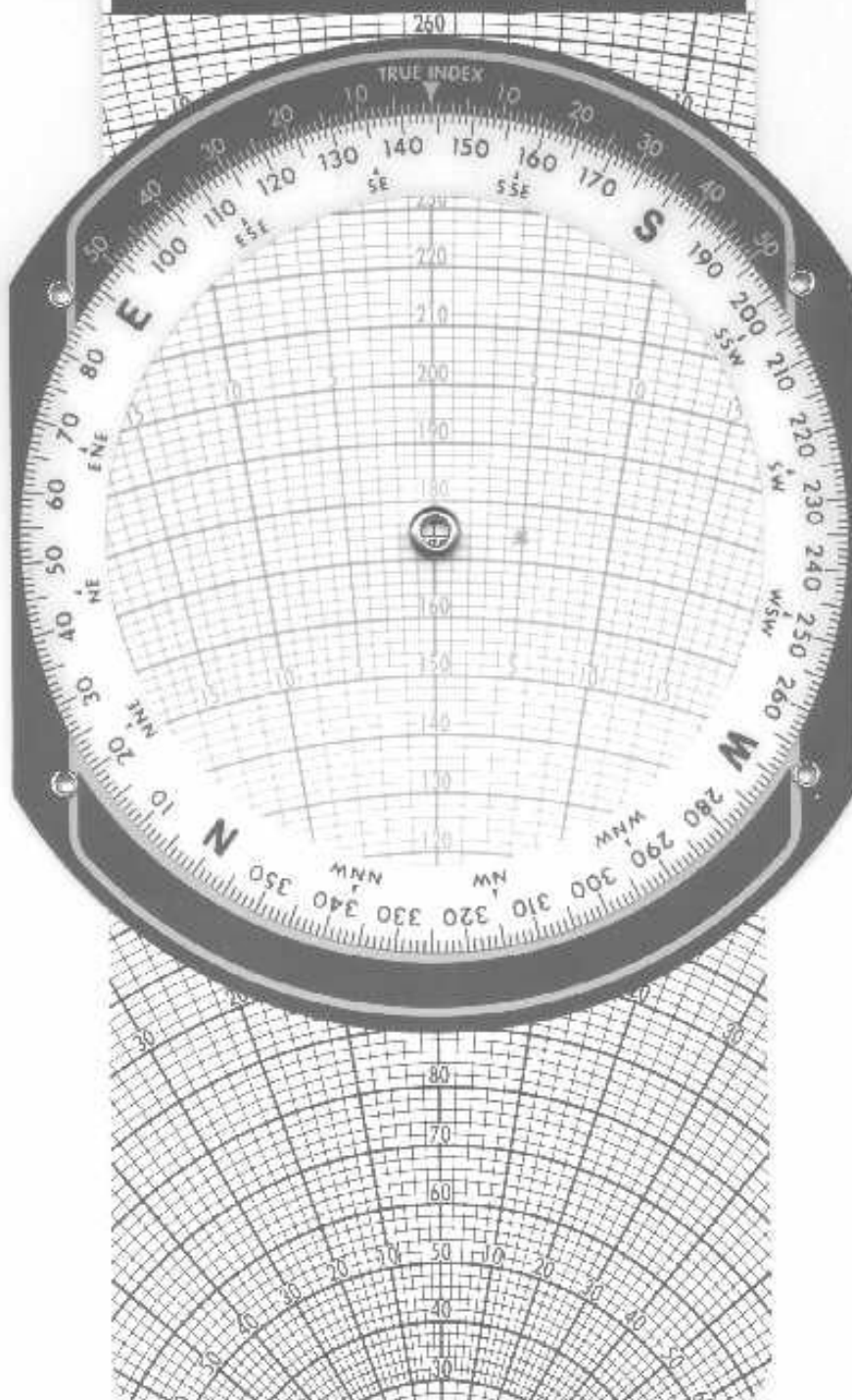
FOR G.S. AND T.H.

- ① PLACE WIND DIRECTION UNDER TRUE INDEX
- ② MARK WIND VELOCITY UP FROM CENTER
- ③ PLACE TRUE COURSE UNDER TRUE INDEX
- ④ SLIDE WIND VELOCITY MARK TO T.A.S. LINE
- ⑤ READ GROUND SPEED UNDER CENTER
- ⑥ READ WIND CORRECTION ANGLE BETWEEN CENTER LINE AND WIND VELOCITY MARK

$$TC \mp \frac{1}{\sin} WCA = TH$$

$$TH \mp \frac{E}{\sin} VAR = MH$$

$$MH \pm DEV = CH$$



Aviation Science

Trigonometric Solutions to a Dead Reckoning Air Navigation Problem using Vector Analysis and Advanced Organizers

Overview: We will complete a dead reckoning navigation problem following certain steps.

Note: Dead reckoning means: Fly a compass heading from a check point for a predetermined period of time. The basic steps are as follows:

1. Draw a straight line between check points selected.
2. Find the true course: Measure the angle between true north and the course line drawn.
3. Convert that true course we drew on the chart to a compass heading we can fly using cockpit instruments e.g. the magnetic compass. We do this by correcting for wind drift, magnetic variation (the difference between true north and magnetic north), and compass deviation using the following formula:

$$TC + R/-L WCA = TH + W/-E VAR = MW +/- Dev. = CH$$

4. Determine the speed we will make good over the ground (Ground Speed) and find the time the flight will take. $Dist. = GS \times Time$.
5. Determine the fuel required. $Fuel\ Req'd = Gal/Hr \times Time$.

Now we will go through the detailed steps we have just discussed.

Given:

1. Airplane True Airspeed (**TAS = 150 kts**).
True Airspeed is the speed through the air which the plane makes good. True airspeed is provided for in the pilot's handbook for the particular aircraft we are using and is dependent on altitude and power setting. True airspeed is independent of wind.
2. Wind Direction and Speed (**240 deg./15 kts.**).
Wind direction and speed will be given in a standard weather briefing prior to the flight. The wind direction is the direction **from** which the wind blows.
3. Fuel Consumption Rate (**15 gal./hr.**).
Fuel consumption rate is provided for in the pilot's handbook for the particular aircraft we are using and is dependent on altitude and power setting.
4. Flight Altitude (**2500 feet**).

The flight altitude is determined by the pilot after careful study of charts and winds. Select an altitude which will keep you at least 1000 feet above any obstacle (2000 feet in mountainous areas). The altitude selected ought to have the most favorable tail wind. The altitude selected should also keep you clear of any altitude restrictions noted on the chart for the route of flight.

5. Route of Flight. (Salisbury, MD to Cambridge, MD)

Select a relatively easy route so that the students might learn and understand the concepts involved in dead reckoning navigation.

Advanced Organizers:

1. Air Navigation Chart

Charts are available free of charge from your State Aviation Administration found in the government section of your telephone directory.

2. *Protractor for measuring angles or PN-1 plotter.

3. *CSG flight navigation calculator.

This flight calculator is not an electronic computer. Buy an inexpensive one made of cardboard and reproduce the calculator parts so that students can make their own flight calculators.

4. *Flight Log.

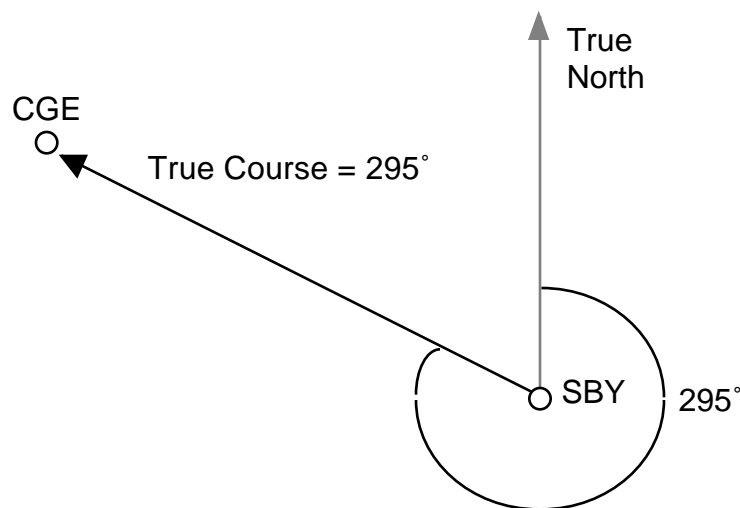
* These items are available at very low cost from Sporty's Pilot Shop (1-800-LIFTOFF) or from the fixed base operator at your local airport. Any pilot you know can help you obtain these items.

Procedures:

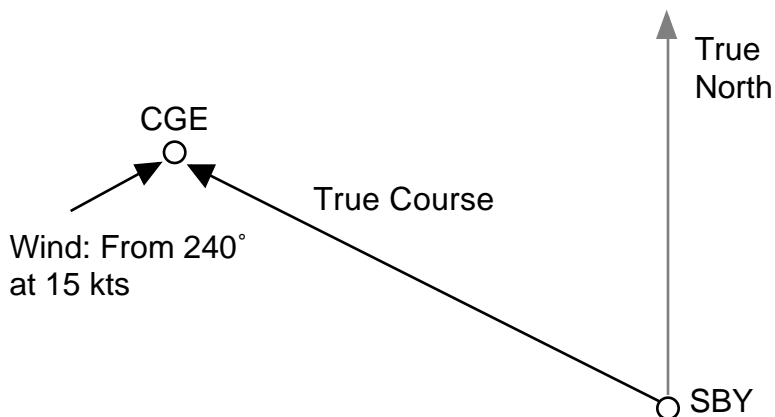
1. Open your air navigation chart and select your departure airport and your destination airport.
2. Draw a straight line from the the center of the runway system at the departure airport to the center of the runway system at the destination airport.
3. **Find the nearest north-south running tick marked line.** This is a line of longitude. Each tick mark represents one minute of latitude. There are 60 minutes per degree of latitude and, of course, 360 degrees around the circumference of the Earth. One minute of latitude also equals (by definition) one nautical mile (NM). Lines of longitude extended northward will pass through the Earth's north pole and will pass through the earth's south pole if extended southward long enough. (With the above information you can determine that there are $60 \times 360 = 21,600$ NM around the circumference of the Earth.)
4. **Measure the angle between the north-south line of longitude and the direction of your flight.** We should get an angle very close to 295 deg. This angle is called the **TRUE COURSE (TC)**. Make sure the angle is correct and makes sense to you based on your direction of flight. Remember that true north (360 deg.) is toward the top of the chart and south (180 deg.) is toward the bottom of the chart. East (090 deg.) is toward the right of the chart and west (270 deg.) is toward the left of the chart. Note too that all courses and headings should be written as three digits so as not to confuse such courses as 300 deg. and 030 deg.!

5. Begin transferring our data to the navigation log (Nav. Log).
 - a. Under the title "Check Points" list the departure airport first followed by the destination airport on the next line down.
 - b. Salisbury has a navigation aid called a "VORTAC". We find the frequency for the VOR (111.2 MHZ) and the three letter Morse code identifier (SBY) listed in the blue box to the southeast of the airport. **List his data in the nav. log under the heading iVORi.**
 - c. Under "Course" write the symbol **D** which means we are flying direct from Salisbury to Cambridge.
 - d. **Place the number "25" in the altitude block** which means our cruise altitude will be 2500 feet as given.
 - e. Place 240/15 in the wind dir./vel. block.
 - f. Write the TAS (150) in the next block.
 - g. In the top block under the symbol "TC" we write in the true course which we measured on the chart, 295.

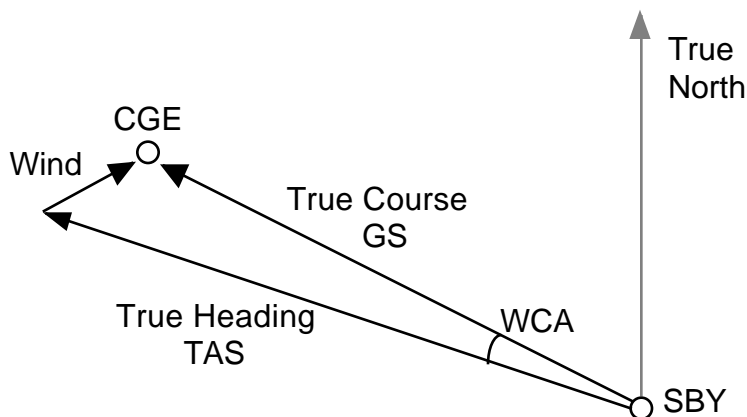
6. This next step will start us on our way toward the vector solution of the triangle shown in fig. 1.
 - a. **We have measured the true course on the chart and we came up with about 295 deg.** This TC is the direction of one of the legs of the triangle. We also need to know the length of that vector which we'll cover shortly.



- b. We can now add the wind dir./vel. vector per fig. 2



- c. Now add the third leg of the triangle which gives us our true heading. The angle between true course and true heading is the wind correction angle (WCA). This means if we turn our airplane into the wind by an angle equal to the WCA, this heading will allow us to 'track' the true course line we drew on the map without being blown off course. See fig. 3



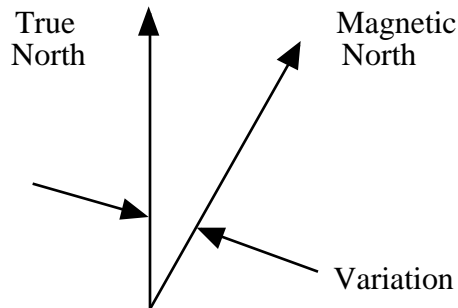
- d. We also see that since we have a headwind component along our route of flight, our speed over the ground will be less than the true airspeed. Therefore the length of the true heading vector is the true airspeed (TAS) and the length of the shorter true course vector will be called the ground speed (GS). We will solve this vector triangle using the wind side of our advanced organizer called the CSG flight computer. The wind side is the side with the movable compass rose on it.

- (1) Follow the directions written at the top of the CSG:

- (a) Place the wind direction (240) under the true index.
- (b) Measure 15 kts up from the center grommet. Mark this location with a dot. Use erasable pencil only. No ballpoint pens! Place a circle around the dot and put a 'W' beside it so you will know it is the wind dot.

- (c) Place the true course (295) under the true index
- (d) Now slide the wind dot so it lies on your TAS line (150 kts)
- (e) Read ground speed (GS) under the center grommet
- (f) Read wind correction angle (WCA) between the centerline and the wind velocity mark.

7. Let us transpose this data to the nav. log.
 - a. Write the WCA (5L) in the block below the TC. Since the WCA is Left, we subtract it from the TC to get True heading (TH).
 - b. Write the TH in the upper block of the next column to the right.
 - c. Write the value of the ground speed under the column marked 'GS'.
8. The next step is to correct the true heading to magnetic heading. The angle between the true north pole and the magnetic north pole at your location is called variation. See fig. 4.



9. We find the actual value of the variation angle on our chart. It is marked by a dashed magenta line. The nearest whole degree of variation closest to our route of flight is 11W.
 - a. Place 11W in the block below the TH on the nav. log. From the nav. log, we see that we add West variation to the TH to get the magnetic heading (MH).
 - b. The next block is for compass deviation (Dev.). Compass deviation is unique to each individual airplane and is the result of compass interference caused by metals and electrons in the vicinity of the wet compass. A compass deviation card is located very near the wet compass so that pilots may correct their magnetic heading for that error. Compass deviation cards are filled out by technicians. A pilot will taxi the airplane out to a large compass rose painted somewhere on the airport surface in an out-of-the way location. The compass rose is aligned with magnetic north. The pilot follows directions from a taxi director on the ground to align the with the various headings of the compass rose. At various headings, the technician will use a non-magnetic screwdriver (brass) to eliminate as much of the error as possible. The error which cannot be eliminated is the compass deviation and is printed on the deviation card.

The card will typically read something like this:

For:	360	030	060	090	120	150	180	210	240	270	300	330
Steer:	001	030	059	088	119	150	181	212	241	270	299	328

Using this example, to fly a magnetic heading of 301 degrees, we would need to fly a compass heading of 300 degrees.

For our purposes, we will assume the deviation is zero because: The computer we use for our cross country simulation does not incorporate a deviation error. The error is zero.

10. The next block on our flight log asks for the distance between the two check points listed. Recalling that each minute of latitude = 1 NM, mark the distance between the check points on a piece of paper and then compare that distance to the number of minutes of latitude along a line of longitude.
11. Ground Speed is determined from the formula:

Distance = Ground Speed (GS = NM/Hr) X Time
12. The fuel required for the trip is determined from the formula:

Fuel required = fuel burn rate (Gal./Hr.) X Time

NOTE: The Federal Aviation Administration requires the following fuel reserves at the point of landing:

Day VFR: 30 minutes of fuel at normal cruise airspeed
Night VFR: 45 minutes of fuel at normal cruise airspeed

The Air force Flight Training Center at Dover AFB requires a minimum of 1 hour of fuel in the tanks at the completion of the flight.

This concludes the flight planning portion of the flight for the purposes of CMST. There is much more information required for preflight preparation than is presented here. For those of you who wish to pursue the subject further, contact your nearest fixed base operator at your local airport or aviation college of your choice. University of Maryland Eastern Shore and other such schools may be found by contacting your state aviation administration.

Solution Key: (for the Salisbury to Cambridge flight outlined above)

True Course: 296°

Ground Speed: 142 kts

Wind Correction: 5° Left

Distance: 28 nm

True Heading: 291°

Magnetic Course: 302°

Estimated time en route: 12 min

Fuel Consumption: 3 gal